

# Probing QCD critical fluctuations from light nuclei production in relativistic heavy-ion collisions

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Large baryon density fluctuations are expected to appear in the matter produced in relativistic heavy-ion collisions if it undergoes a first-order phase transition from the quark-gluon plasma to the hadronic matter. In the case that the density fluctuations can survive final-state interactions during the hadronic evolution and persist until the kinetic freeze-out, they then provide a unique probe to the critical endpoint (CEP), at which the first-order phase transition changes to a smooth crossover, in the QCD phase diagram. In the present study, we demonstrate for the first time that information on the neutron relative density fluctuation  $\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$  at freeze-out can be obtained from the yield ratio of light nuclei, i.e.,  $\mathcal{O}_{p-d-t} = N_{3H}N_p/N_d^2$ , in heavy-ion collisions. From recent results on the  $p$ ,  $d$  and  $^3H$  yields in central Pb+Pb collisions at the CERN Super Proton Synchrotron (SPS) energies measured by the NA49 Collaboration, we observe a non-monotonic behavior of  $\Delta n$  as a function of the collision energy with a peak at  $\sqrt{s_{NN}} = 8.8$  GeV, suggesting that the QGP produced at this collision energy may have passed through the CEP during its evolution.

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The most important goal of relativistic heavy-ion collisions, including those being carried out at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC), is to study the properties of produced quark-gluon plasma (QGP) and its transition to hadronic matter [1, 2]. Results of lattice quantum chromodynamics (LQCD) simulations [3–8] and effective model approaches [9–12] have indicated that the quark-hadron transition at large values of baryon chemical potentials  $\mu_B$  is likely a first-order phase transition, while the transition at small  $\mu_B$  is a crossover, suggesting the existence of a critical endpoint (CEP) where the first-order phase transition ends. Searching for the CEP and finding its location in the temperature versus baryon chemical potential ( $T, \mu_B$ ) plane of QCD phase diagram is currently one of the most active research areas in relativistic heavy-ion physics. It has been argued that the enhanced long-wavelength density fluctuations near the CEP can lead to singularities in all thermodynamic observables [2]. The resulting event-by-event fluctuations of conserved quantities in relativistic heavy-ion collisions have thus played a central role in both theoretical and experimental searches for the CEP, and one can see Ref. [13] for an overview of recent progresses.

The properties of the matter formed in relativistic

heavy-ion collisions depend on the collision energy and centrality. For certain collision energy and centrality, the temperature and baryon chemical potential of produced matter at phase transition may be at the CEP of QCD phase diagram, and when this happens, one expects a non-monotonic behavior in the collision energy and centrality dependence of some properties of the matter, such as the ratio of its shear viscosity to entropy density [14, 15] and expansion speed [16, 17]. Recently, a non-monotonic excitation function for the Gaussian emission source radii difference extracted from two-pion interferometry measurements [18–20] in Au+Au ( $\sqrt{s_{NN}}=7.7$ –200 GeV) and Pb+Pb ( $\sqrt{s_{NN}}=2.76$  TeV) collisions has been observed with a maximum value located around  $\sqrt{s_{NN}} = 40$  GeV [21]. In addition, the energy dependence of the fourth-order fluctuation ( $\kappa\sigma^2$ ) of net-proton distribution measured in the Beam Energy Scan (BES) program by the STAR Collaboration exhibits a largest deviation from unity in Au+Au collisions at  $\sqrt{s_{NN}} = 7.7$  GeV [22]. Although it remains a great challenge to unambiguously relate these observed non-monotonic behaviors of physical quantities to the CEP, studying all such non-monotonic behaviors in present and future experiments is expected to lead to a definitive answer on whether the CEP exists and its location in the QCD phase diagram.

In analogy to the critical opalescence observed in the liquid-gas phase transition [23, 24], the matter created in relativistic heavy-ion collisions would develop large baryon density fluctuations when its evolution trajectory in the ( $T, \mu_B$ ) plane of QCD phase diagram passes near the CEP or across the first-order phase transition line.

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Studies based on both the hydrodynamic approach [25–27] and the microscopic transport model [28] indeed show that the spinodal instability due to the first-order phase transition can induce large baryon density fluctuations, which could lead to strong nucleon density fluctuations at kinetic freeze-out in relativistic heavy-ion collisions.

In this Letter, we show for the first time that the relative density fluctuation of neutrons ( $\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$ ) at freeze-out in relativistic heavy-ion collisions can be encoded in the yield ratio of light nuclei, namely,  $\mathcal{O}_{p-d-t} = N_{3H}N_p/N_d^2$ . Our study is based on the coalescence model [29–38], which has recently been shown to describe very well the experimental data from both RHIC and LHC [39]. In particular, we use a newly developed analytical formula for cluster production in the coalescence model [40]. In this model, the formation of light nuclei depends on the nucleon many-body correlations and is thus affected by the fluctuations in the nucleon number or density. More quantitatively, we observe a peak of  $\Delta n$  in Pb+Pb collisions at  $\sqrt{s_{NN}} = 8.8$  GeV from analyzing the very recent results on the proton ( $p$ ), deuteron ( $d$ ) and triton ( $t$  or  ${}^3\text{H}$ ) yields in Pb+Pb collisions at SPS energies measured by the NA49 Collaboration [41]. Our result thus has the advantage of directly measuring the density fluctuation instead of using the number fluctuation to infer the density fluctuation as having done so far. Furthermore, our result allows us to estimate that the temperature and baryon chemical potential at which the CEP is located in the QCD phase diagram are  $T^{\text{CEP}} \sim 144$  MeV and  $\mu_B^{\text{CEP}} \sim 385$  MeV.

We start by briefly introducing the newly derived analytical coalescence formula COAL-SH [40] for cluster production in relativistic heavy-ion collisions. In COAL-SH, the yield  $N_c$  per unit rapidity of a cluster at midrapidity and consisting of  $A$  constituent particles from the hadronic matter at kinetic freeze-out or emission source of an effective temperature  $T_{\text{eff}}$  including the effect of transversal flow, volume  $V$ , and number  $N_i$  and mass  $m_i$  of the  $i$ -th constituent reads [40]

$$N_c = g_{\text{rel}} g_{\text{size}} g_c M^{3/2} \left[ \prod_{i=1}^A \frac{N_i}{m_i^{3/2}} \right] \times \prod_{i=1}^{A-1} \frac{(4\pi/\omega)^{3/2}}{Vx(1+x^2)} \left( \frac{x^2}{1+x^2} \right)^{l_i} G(l_i, x). \quad (1)$$

In the above,  $M = \sum_{i=1}^A m_i$  is the rest mass of the cluster,  $l_i$  is the orbital angular momentum associated with the  $i$ -th relative coordinate,  $\omega$  is the oscillator frequency of the cluster's internal wave function and is inversely proportional to  $Mr_{\text{rms}}^2$  with  $r_{\text{rms}}$  being the root-mean-square (RMS) radius of the cluster, and  $G(l, x) = \sum_{k=0}^l \frac{l!}{k!(l-k)!} \frac{1}{(2k+1)x^{2k}}$  with  $x = (2T_{\text{eff}}/\omega)^{1/2}$  is the suppression factor due to orbital angular momentum on the coalescence probability [42]. In addition,  $g_c = (2S+1)/(\prod_{i=1}^A (2s_i+1))$  is the coalescence fac-

tor for constituents of spin  $s_i$  to form a cluster of spin  $S$ ,  $g_{\text{rel}}$  is the relativistic correction to the effective volume in momentum space, and  $g_{\text{size}}$  is the correction due to the finite size of produced cluster.

In central Pb+Pb collisions considered here,  $V$  is much larger than the size of light nucleus, and we thus set  $g_{\text{size}} = 1$ . We also set  $g_{\text{rel}} = 1$  since the masses of nucleons and light nuclei are much larger than the value of  $T_{\text{eff}}$ . For light nuclei included in the present study, all the constituent nucleons are in  $s$ -state ( $l = 0$ ), and we thus have  $G(l, x) = 1$ . The yields of  $d$  and  ${}^3\text{H}$  are then simply given by

$$N_d = g_d \frac{(m_n + m_p)^{3/2}}{m_p^{3/2} m_n^{3/2}} \frac{N_p N_n}{V} \frac{(4\pi/\omega_d)^{3/2}}{x_d(1+x_d^2)}, \quad (2)$$

$$N_{3H} = g_{3H} \frac{(2m_n + m_p)^{3/2}}{m_p^{3/2} m_n^3} \frac{N_p N_n^2}{V^2} \frac{(4\pi/\omega_{3H})^3}{x_{3H}^2(1+x_{3H}^2)^2}, \quad (3)$$

where  $N_p$  ( $N_n$ ) is the number of protons (neutrons) in the emission source, the coalescence factor is  $g_d = 3/4$  for  $d$  and  $g_{3H} = 1/4$  for  ${}^3\text{H}$ , and we denote  $x_d = (2T_{\text{eff}}/\omega_d)^{1/2}$  and  $x_{3H} = (2T_{\text{eff}}/\omega_{3H})^{1/2}$  with the oscillator frequency  $\omega_d = 8.1$  MeV for  $d$  and  $\omega_{3H} = 13.4$  MeV for  ${}^3\text{H}$  obtained from their respective RMS radii  $r_{\text{rms},d} = 1.96$  fm and  $r_{\text{rms},3H} = 1.76$  fm [43]. The effective temperature  $T_{\text{eff}}$  at the kinetic freeze-out in relativistic heavy-ion collision is typically about 200 MeV and is thus much larger than the oscillator frequencies  $\omega_d$  and  $\omega_{3H}$ . Neglecting proton and neutron mass difference ( $m_p = m_n = m_0$ ) and noting  $x_d, x_{3H} \gg 1$ , we then have

$$N_d = \frac{3}{2^{1/2}} \left( \frac{2\pi}{m_0 T_{\text{eff}}} \right)^{3/2} \frac{N_p N_n}{V}, \quad (4)$$

$$N_{3H} = \frac{3^{3/2}}{4} \left( \frac{2\pi}{m_0 T_{\text{eff}}} \right)^3 \frac{N_p N_n^2}{V^2}. \quad (5)$$

We note that although the coalescence formula COAL-SH is derived by assuming the Bjorken boost invariance [44], Eqs.(4) and (5) are also valid for an isotropically expanding fireball. This is not surprising as only its effective temperature, volume, and proton and nucleon numbers appear in these equations. Also, the above equations are consistent with the predictions from the thermal (statistical) model [45–48] if  $p$ ,  $n$ ,  $d$  and  ${}^3\text{H}$  are assumed to be in thermal and chemical equilibrium and the binding energies of  $d$  and  ${}^3\text{H}$  are neglected.

In obtaining Eqs. (4) and (5), we have assumed that nucleons are uniformly distributed in space at freeze-out. To include density fluctuation of nucleons, we take the neutron density in the emission source to be

$$n(\vec{r}) = \frac{1}{V} \int n(\vec{r}') d\vec{r}' + \delta n(\vec{r}) = \langle n \rangle + \delta n(\vec{r}), \quad (6)$$

where  $\langle \cdot \rangle$  denotes the average over space and  $\delta n(\vec{r})$  with  $\langle \delta n \rangle = 0$  denotes the fluctuation of neutron density from

TABLE I: The yields ( $dN/dy$  at midrapidity) of  $p$ ,  $d$ ,  ${}^3\text{He}$  and  ${}^3\text{H}$  as well as the yield ratio  ${}^3\text{H}/{}^3\text{He}$  measured in Pb+Pb collisions at SPS energies [41] together with the derived yield ratio  $\mathcal{O}_{p-d-t}$  and the neutron relative density fluctuation  $\Delta n$ . The units for  $E$  and  $\sqrt{s_{NN}}$  are AGeV and GeV, respectively.

$E$	$\sqrt{s_{NN}}$	centrality	$p$	$d$	${}^3\text{He}$	${}^3\text{H}/{}^3\text{He}$	${}^3\text{H}$	$\mathcal{O}_{p-d-t}$	$\Delta n$
20	6.3	0 – 7%	$46.1 \pm 2.1$	$2.094 \pm 0.168$	$3.58(\pm 0.43) \times 10^{-2}$	$1.22 \pm 0.10$	$4.37(\pm 0.64) \times 10^{-2}$	$0.459 \pm 0.014$	$0.583 \pm 0.048$
30	7.6	0 – 7%	$42.1 \pm 2.0$	$1.379 \pm 0.111$	$1.89(\pm 0.23) \times 10^{-2}$	$1.18 \pm 0.11$	$2.23(\pm 0.34) \times 10^{-2}$	$0.494 \pm 0.020$	$0.704 \pm 0.068$
40	8.8	0 – 7%	$41.3 \pm 1.1$	$1.065 \pm 0.086$	$1.28(\pm 0.15) \times 10^{-2}$	$1.16 \pm 0.15$	$1.48(\pm 0.26) \times 10^{-2}$	$0.541 \pm 0.022$	$0.864 \pm 0.076$
80	12.3	0 – 7%	$30.1 \pm 1.0$	$0.543 \pm 0.044$	$3.90(\pm 0.50) \times 10^{-3}$	$1.15 \pm 0.19$	$4.49(\pm 0.94) \times 10^{-3}$	$0.458 \pm 0.038$	$0.579 \pm 0.130$
158	17.3	0 – 12%	$23.9 \pm 1.0$	$0.279 \pm 0.023$	$1.50(\pm 0.20) \times 10^{-3}$	$1.05 \pm 0.15$	$1.58(\pm 0.31) \times 10^{-3}$	$0.484 \pm 0.037$	$0.668 \pm 0.127$

its average value  $\langle n \rangle$ . Neglecting the correlations between proton and neutron density fluctuations at freeze-out, we can then rewrite Eqs. (4) and (5) as

$$N_d = \frac{3}{2^{1/2}} \left( \frac{2\pi}{m_0 T} \right)^{3/2} N_p \langle n \rangle, \quad (7)$$

$$N_{3H} = \frac{3^{3/2}}{4} \left( \frac{2\pi}{m_0 T} \right)^3 N_p \langle n \rangle^2 (1 + \Delta n), \quad (8)$$

where  $\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$  is a dimensionless quantity reflecting the relative density fluctuation of neutrons. Eqs. (7) and (8) clearly show that the neutron density fluctuation  $\langle (\delta n)^2 \rangle$  always tends to increase the yield of  ${}^3\text{H}$  although it does not affect the yield of  $d$ .

Besides  $\Delta n$ , the yield of  ${}^3\text{H}$  also depends on  $T$ ,  $N_p$  and  $\langle n \rangle$ . In order to cleanly probe the density fluctuation in the emission source, we consider the following yield ratio:

$$\mathcal{O}_{p-d-t} = \frac{N_{3H} N_p}{N_d^2} = g(1 + \Delta n), \quad (9)$$

with  $g = 4/9 \times (3/4)^{3/2} \approx 0.29$ .  $\mathcal{O}_{p-d-t}$  is constructed in such a way that many effects, such as those due to  $T$ ,  $N_p$ ,  $\langle n \rangle$ , volume and isospin asymmetry of the emission source, cancel out. It is seen that  $\mathcal{O}_{p-d-t}$  has a very simple linear dependence on  $\Delta n$ . Experimentally, one can extract  $\Delta n$  in relativistic heavy-ion collisions by measuring the yield ratio  $\mathcal{O}_{p-d-t}$ . We would like to point out that one may also choose other light nuclei such as  ${}^3\text{He}$  and  ${}^4\text{He}$  to extract the nucleon density fluctuation at kinetic freeze-out. In these cases, information on the isospin at freeze-out is, however, needed and also the higher-order density fluctuations may be involved. For example, the yield of  ${}^4\text{He}$  is given by

$$N_{4He} = \frac{1}{2} \left( \frac{2\pi}{m_0 T} \right)^{9/2} N_p \langle n_p \rangle \langle n \rangle^2 (1 + \Delta n + \Delta n_p), \quad (10)$$

which further depends on the proton density  $\langle n_p \rangle$  and its relative density fluctuation  $\Delta n_p = \langle (\delta n_p)^2 \rangle / \langle n_p \rangle^2$ .

Eqs. (8) and (10) show that large density fluctuations would increase the yields of light nuclei in relativistic heavy-ion collisions and lead to an  $A$  dependence different from  $\langle n \rangle^A$  as expected from the statistical model

prediction [49]. The existing experimental data from the Alternating Gradient Synchrotron (AGS) at  $\sqrt{s_{NN}} = 4.8$  GeV have shown striking exponential behavior with a penalty factor of about 50 per additional nucleon to the produced nuclear cluster up to  $A = 7$  [49]. Similarly, such a regular exponential behavior is seen at RHIC energies for  $A \leq 4$  [50]. These results have thus ruled out large nucleon density fluctuations at freeze-out in heavy-ion collisions at AGS and RHIC top energies.

On the other hand, recently published results of light nucleus production in central Pb+Pb collisions at SPS energies [41] show a quite different behavior. This can be seen from the collision energy dependence of  $\mathcal{O}_{p-d-t}$  and  $\Delta n$ . Table I summarizes the yields ( $dN/dy$  at midrapidity) of  $p$ ,  $d$ ,  ${}^3\text{He}$  and  ${}^3\text{H}$  as well as the yield ratio  ${}^3\text{H}/{}^3\text{He}$  measured in central Pb+Pb collisions at 20 AGeV (0 – 7% centrality), 30 AGeV (0 – 7% centrality), 40 AGeV (0 – 7% centrality), 80 AGeV (0 – 7% centrality), and 158 AGeV (0 – 12% centrality) from the NA49 Collaboration [41]. In obtaining the yield of  ${}^3\text{H}$ , we have used the relation  ${}^3\text{H} = {}^3\text{He} \times {}^3\text{H}/{}^3\text{He}$ . The derived  $\mathcal{O}_{p-d-t}$  and  $\Delta n$  are also included in Table I with the errors estimated by assuming they are dominated by correlated systematic errors as a result of similar detector acceptance and phase-space extrapolation. Very interestingly, it is seen from Table I that the energy dependence of  $\mathcal{O}_{p-d-t}$  exhibits a non-monotonic behavior and reaches its largest value at  $\sqrt{s_{NN}} = 8.8$  GeV. A similar non-monotonic behavior is seen in the dependence of  $\Delta n$  on the collision energy, as expected from Eq.(9). We note that this value of the neutron relative density fluctuation is much larger than that due to the event-by-event statistical fluctuation in the neutron multiplicity, which is expected to be inversely proportional to its mean value and is thus only about a few per cent.

To see more clearly the collision energy dependence of  $\Delta n$ , we plot in Fig. 1 the extracted  $\Delta n$  as a function of  $\sqrt{s_{NN}}$ . It shows that  $\Delta n$  reaches the largest value in central Pb+Pb collisions at  $\sqrt{s_{NN}} = 8.8$  GeV. For central Pb+Pb collisions at higher incident energies (e.g.,  $\sqrt{s_{NN}} = 12.3$  GeV and 17.3 GeV), the reaction system may undergo a crossover rather than a first-order phase transition between the QGP and the hadronic matter,

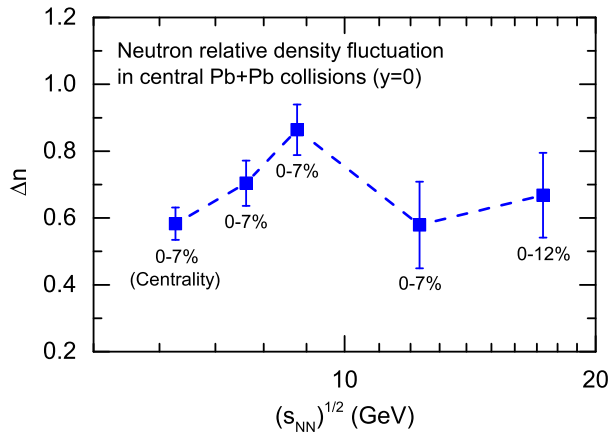


FIG. 1: Collision energy dependence of the neutron relative density fluctuation  $\Delta n$  in central Pb+Pb collisions at SPS energies based on data from Ref. [41].

and the density fluctuation in the produced matter is thus insignificant. With decreasing incident energy (e.g., around  $\sqrt{s_{NN}} = 8.8$  GeV), the reaction system may pass through the CEP and develop the largest density fluctuation. With further decrease in the incident energy (e.g., at  $\sqrt{s_{NN}} = 6.3$  GeV and 7.6 GeV), the reaction system may barely move near the first-order transition line, so only a relatively small density fluctuation is induced. When the incident energy is further lowered, the reaction system may miss the first-order transition line and no quark-hadron phase transition occurs in the collisions, thus resulting in negligible density fluctuation at the kinetic freeze-out. The slightly larger  $\Delta n$  at  $\sqrt{s_{NN}} = 17.3$  GeV than at 12.3 GeV could be due to the larger centrality at  $\sqrt{s_{NN}} = 17.3$  GeV which leads to a larger  $g$  in Eq. (9). Therefore, the non-monotonic behavior shown in Fig. 1 is consistent with the scenario that the CEP may be reached by the produced QGP during its time evolution in central Pb+Pb collisions around  $\sqrt{s_{NN}} = 8.8$  GeV. From the parametrization in Ref. [51] for the chemical freeze-out conditions based on the statistical model fit to available experimental data, the temperature and baryon chemical potential at  $\sqrt{s_{NN}} = 8.8$  GeV are estimated to be  $T \sim 144$  MeV and  $\mu_B \sim 385$  MeV. It is interesting to note that the estimated  $\mu_B \sim 385$  MeV for CEP is close to those predicted from the LQCD [6] and Dyson-Schwinger equation (DSE) [52] as well as that based on the hadronic bootstrap approach [53]. Also, the collision energy  $\sqrt{s_{NN}} = 8.8$  GeV corresponds to that at which a peak is seen in the measured  $K^+/\pi^+$  ratio by the NA49 Collaboration [54], which has been interpreted as a signature for the onset of QGP formation [55] or the restoration of chiral symmetry [56] in these collisions.

Although the present study is based on the simple formulas in Eq. (4) and Eq. (5), the non-monotonic behavior in the relative neutron density fluctuation extracted

from the measured yield ratio  $\mathcal{O}_{p-d-t}$  will still be present if the more accurate formula in Eq.(1) is used. This is because the latter will increase the value of  $g$  in Eq.(9) by less than 50%, for which the peak of  $\Delta n$  remains at  $\sqrt{s_{NN}} = 8.8$  GeV. Even assuming that the value of  $g$  increases linearly with decreasing  $\sqrt{s_{NN}}$ , such a non-monotonic behavior is still seen.

In summary, with a newly derived analytical coalescence formula for cluster production in heavy-ion collisions, we have demonstrated that information on the relative density fluctuation of neutrons ( $\Delta n = \langle(\delta n)^2\rangle/\langle n\rangle^2$ ) at the kinetic freeze-out can be determined from the yield ratio  $\mathcal{O}_{p-d-t} = N_{3H}N_p/N_d^2$ . From measured yields of light nuclei at SPS energies by the NA49 Collaboration, we have extracted the collision energy dependence of  $\Delta n$  and found that the  $\Delta n$  exhibits a non-monotonic behavior with a peak at  $\sqrt{s_{NN}} = 8.8$  GeV, suggesting that the CEP in the QCD phase diagram may have been reached in these collisions with its temperature and baryon chemical potential estimated to be  $T^{\text{CEP}} \sim 144$  MeV and  $\mu_B^{\text{CEP}} \sim 385$  MeV, respectively. Although this circumstantial evidence is quite interesting and striking, our study is based on a simplified theoretical model and one set of experimental data. To establish our approach as a viable tool in the search of the QCD critical endpoint requires further investigations from experiments, such as the BES program at RHIC in this energy range with high luminosity beams as well as detectors of excellent particle identification and large acceptance, and theoretical modeling of nucleus production and its connection to baryon density fluctuations.

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